A Glass Always Half Full: Reconsideration of the Wales Apparatus to Apply Constant Head Boundary Conditions

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Abstract

A new apparatus is presented that is capable of applying a constant fluid pressure at inflow and outflow boundaries. The apparatus can be refilled during operation and does not rely on an overflow mechanism. The device is constructed of two vessels, one that contains the delivered fluid and the other that contains a less dense fluid. By matching the fluid densities and the areas of the vessels, the absolute elevation of the delivered fluid is maintained as the fluid is added to or removed from the system. The history of the development of the device, the underlying physical principles, and two demonstrations of the operation of a prototype device are shown.

Introduction

There are few things upon which almost everyone will agree. One is that if you remove water from a vessel, the water level will fall. Or will it? This question lies at the heart of a brief, inconclusive, debate that flared over 70 years ago regarding devices that can be used to supply fluid flow at a constant rate or pressure. Being a requisite boundary condition for many hydraulic experiments, the relevance of such a system has not diminished in the intervening years.

In a rapid exchange of articles, J.H. Wales (1934) suggested that a twin vessel apparatus [Figure 1, bottom left] would be a feasible device to supply water at a constant flux. This prompted a reply by McCarthy (1934), suggesting that a Mariotte bottle [Figure 1, top left] was a simpler, existing design. McCubbin (1934) further criticized Wales' design as overcomplex and stated, without proof, that it would not work. Finishing the thread of articles, Pierce (1934) suggested a somewhat elaborate overflow container [Figure 1, top right] to provide constant inflow. While Mariotte bottles and simple overflow devices are used widely in disciplines ranging from hydrology to medical research, Wales' design appears to have been forgotten, with the physics left unresolved.

The objectives of this paper are to review the historical debate and clarify the physics of the Wales apparatus. Based on this explanation, we suggest that the device should be considered for more widespread use in hydrology and related disciplines.

Theory

Wales' device can be modified to provide a very useful and novel apparatus [Figure 1, bottom right]. The idea underlying the modified Wales apparatus is as follows: fluid is removed from a floating inner vessel, causing the level within the inner vessel to fall; but, the reduction of the mass of the inner vessel causes it to float higher in the outer vessel. If the sizes of the vessels and the densities of the fluids are matched, the absolute elevation of the inner fluid will remain constant as fluid is removed (or added). Specifically, consider an apparatus constructed of two circular cylinders, both sealed at the bottom and open at the top, as shown by Wales. The outer cylinder (Wales' "B") has an inner cross sectional area, A_B. A fixed volume, V_1, of fluid (Wales' "1") with density ρ_1 is added to the outer cylinder. The outer cross sectional area of the inner cylinder is A_{A_out} and the inner cross sectional area of the inner cylinder is A_{A_in}. The total mass of the inner cylinder, when empty, is M_A'. Initially, a volume, V_2, of a second fluid (Wales' "2") with density, ρ_2, is added to the inner cylinder. The height of the inner fluid above the base of the outer cylinder, z_2, will be:

$$z_2 = \frac{V_1}{A_B} + \frac{A_{A\_out} H_{below}}{A_B} - H_{below} + \frac{V_2}{A_{A\_in}} \tag{1}$$

where H_{below} is the depth of the base of the inner cylinder below the level of the outer fluid. H_{below} will adjust until the mass of the inner cylinder equals the mass of the outer fluid displaced by the inner cylinder.

$$H_{below} A_{A\_out} \rho_1 = V_2 \rho_2 + M_A' \tag{2}$$

This leads to an expression for the elevation of the inner fluid above the base of the outer cylinder, z_2:

$$z_2 = \frac{V_1}{A_B} + \frac{V_2 \rho_2 + M_A'}{A_B \rho_1} - \frac{V_2 \rho_2 + M_A'}{A_{A\_out} \rho_1} + \frac{V_2}{A_{A\_in}} \tag{3}$$

For the absolute elevation of the inner fluid to remain constant as the volume of the inner fluid changes, $dz_2/dV_2 = 0$. This requires that:

$$\frac{\rho_1}{\rho_2} = A_{A\_in}\left(\frac{1}{A_{A\_out}} - \frac{1}{A_B}\right) \tag{4}$$

If the inner container has thin walls, then $A_{A\_in} \approx A_{A\_out} = A_A$, leading to:

$$A_A = \left(1 - \frac{\rho_1}{\rho_2}\right) A_B \tag{5}$$

The preceding analysis demonstrates that Wales' design, which explicitly states that water be used in containers A and B ($\rho_1 = \rho_2$), would not work, as suggested by Pierce. A further practical limitation is clear if we consider Wales' figure to represent cylinders in cross section with water in the inner container. Then, the required density of the outer fluid can be determined to be 400 kg/m^3 using the cross sectional areas and equation 5. This is far lower than most readily available fluids. A more practical design would use a readily available outer fluid such as mineral oil (e.g. heavy mineral oil, with a density of 860 kg/m^3 at 25^oC). To deliver water from the inner cylinder with mineral oil in the outer cylinder, the outer cylinder must have a diameter that is 2.67 times that of the inner cylinder. As a result, the simple design of two cylinders envisioned by Wales is generally unworkable because the small diameter inner container will not remain vertical within the larger diameter outer container. (Stabilizing fins could be added to the inner cylinder, but this leads to a more complex design to construct.) To address this limitation, the outer reservoir can be replaced with two vessels (B_1 and B_2), which are plumbed together (Figure 2). Vessel B_1 can be replaced with several connected bottles; only their cumulative cross sectional area is important. This is shown in the top-view inset of Figure 2 as vessels with areas A_{B1} and A_{B2}.

Methods

To test the modified Wales device, we constructed a prototype using readily available materials: three 2-liter soda-bottles, one 0.591 liter soda-bottle, a section of 0.0127 m ID transparent PVC tubing, rigid plastic irrigation pipe and connectors, and flexible plastic tubing (Figure 1, lower right). The 2-liter bottles were inverted, punctured at their bases to allow air entry, and plumbed together using the irrigation pipe with standard hose fittings. This reservoir was connected to the base of the capped PVC tube by flexible plastic tubing. The total cross sectional area of the three 2-liter bottles and the PVC tube is 0.0098 m^2. A 0.591 liter bottle, with its bottom cut off, was inverted within the PVC tube. The close fit of the floating bottle in the PVC tube kept the inner cylinder vertical, while allowing free movement of the bottle as the inner cylinder floats or sinks in the outer fluid. That is, in the photograph of a modified Wales apparatus in Figure 1 (bottom right), vessel B_2 is a clear cylinder; Wales' vessel A is an inverted bottle that fits closely within vessel B_2. To provide fluids with the same density contrast as a mineral oil – water system while avoiding complications related to handling and disposal of oil and to allow for more precise control of the fluid density contrast, water was used as the outer fluid and a sugar solution (ρ_i = 1150 kg/m^3 at 25°C) was used as the inner fluid. The inner fluid was dyed with food coloring for visibility. The addition of a siphon to draw water from the inner vessel (shown in Figure 2, but not shown in Figure 1, lower right) will only affect the absolute elevation of the vessel, it will not affect the stability of the inner fluid level during operation.

Two demonstrations of the operation of the modified Wales device are presented. First, a series of photographs are shown to demonstrate that the elevation of the top of the fluid in the inner vessel is constant as fluid is removed from the inner vessel. Second, the volumetric flow rate is shown as fluid is removed from the inner vessel through a siphon hose, as shown in Figure 2. Specifically, the elapsed time is shown as increments of 10 ml of fluid flowed from the inner vessel. After 100 ml of fluid was removed, fluid was poured manually into the inner vessel, without interrupting the outflow. The outflow rate was monitored continuously until 200 ml of fluid was collected.

Results

The photographs in Figure 1, lower right, show the inner container as fluid is removed in two stages. That is, the position of the inner vessel is shown in the outer vessel for three different volumes of inner fluid. Comparing the leftmost and rightmost photographs, the volume of fluid in the inner container is reduced by 0.306 l, the fluid level within the inner container changes by 0.076 m relative to the base of the inner container, but the elevation of the fluid in the inner container changes by less than 0.001 m, verifying the simple hydrostatic basis of the design.

The device maintained a constant rate of outflow of 0.06 ml/s throughout the outflow experiment ($R^2 = 0.9998$, Figure 3). Furthermore, the rate was constant during both removal and filling of fluid. This demonstrates the operation of the device and our fundamental assertion that it can be refilled during operation. Given its ability to maintain a constant elevation of the level of the fluid in the inner vessel during addition or removal of fluid, the device could also be configured to provide a constant head inflow or outflow boundary conditions.

Conclusions

Unlike a Mariotte bottle, the modified Wales device is refillable without interruption of an experiment. Therefore, it can be used to provide a constant water energy potential at inflow or outflow boundaries as well as providing a constant flux inflow condition. Unlike an overflow device such as suggested by Pierce (1934), it does not require constant pumping. Additionally, the modified Wales apparatus requires a smaller reservoir of fluid than a Mariotte bottle, if the fluid is re-circulated, and it requires less fluid than an overflow apparatus, which must have excess flow provided to maintain an overflow condition. Marriott bottles and overflow devices are clearly preferable for applications for which the delivered fluid is readily available (overflow) or for which the total amount of fluid to be delivered is relatively small (Mariotte). But, the modified Wales apparatus may have advantages if it is important to conserve the delivered fluid or if experiments are to continue for a long period of time. Furthermore, the Wales apparatus can provide a constant head outflow boundary condition.

Acknowledgements

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References

J. H. Wales, **Device for Constant Flow of Liquids**, *Science* **79**, 545 (1934).

E. L. McCarthy, **Mariotte's Bottle**, *Science* **80** 100 (1934).

W. A. McCubbin, **An Over-Complex Apparatus**, *Science* **80** 144 (1934).

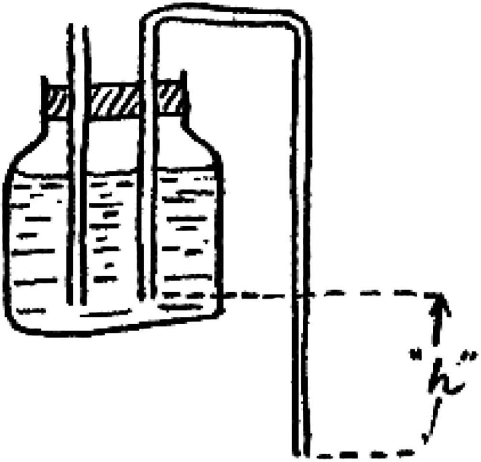
H. F. Pierce, **A Constant-Rate Dropping Device for Liquids**, *Science* **80** 339 (1934).

Figure Captions:

Figure 1. top left) Mariotte bottle described by McCarthy; top right) overflow bottle described by Pierce; bottom left) Double reservoir apparatus suggested by Wales; center); and bottom right) Modified Wales apparatus showing the reservoir with three different volumes of inner dyed fluid.

Figure 2. Schematic diagram of modified Wales' apparatus shown with a siphon to deliver the inner fluid at a constant rate. Wales' vessel "B" has been divided into a reservoir with area A_{B1} and a smaller diameter tube with area A_{B2}; the total area of the outer vessel is $A_B = A_{B1} + A_{B2}$.

Figure 3. Elapsed time, seconds, as a function of cumulative volume of outflow, ml, for an apparatus configured as shown in Figure 2. Fluid was poured manually into the inner vessel after 100 ml of outflow was collected (shown as a vertical dashed line). Outflow was continuous throughout the experiment.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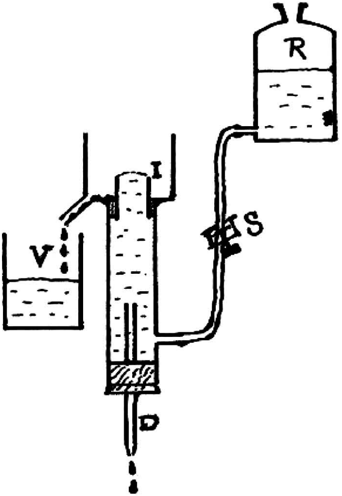
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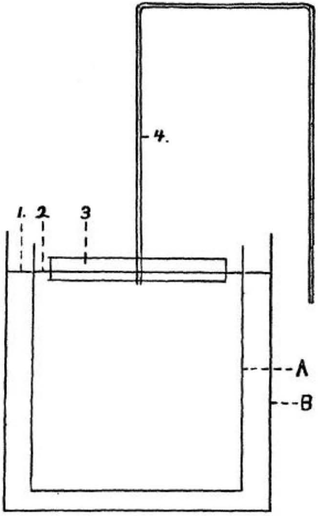
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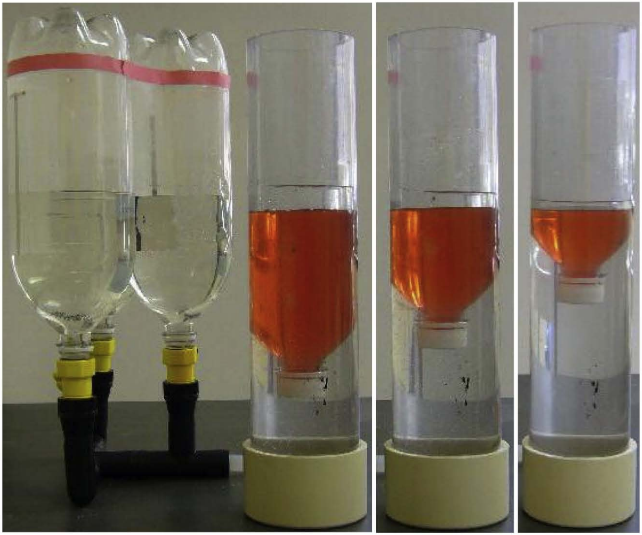

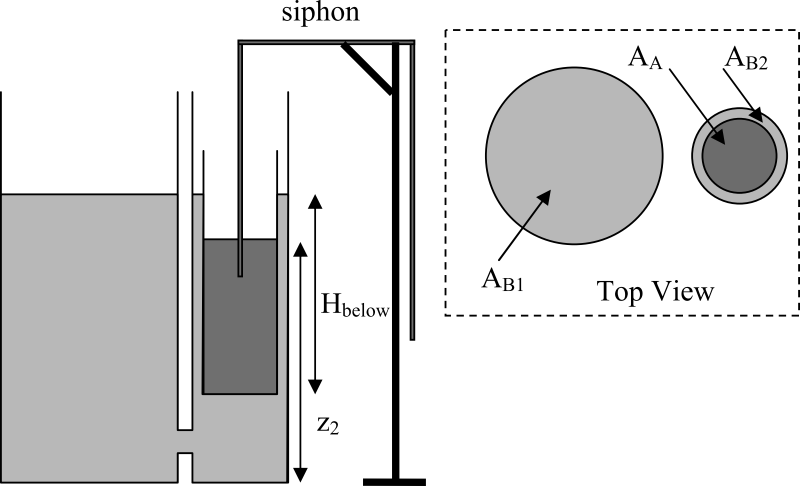
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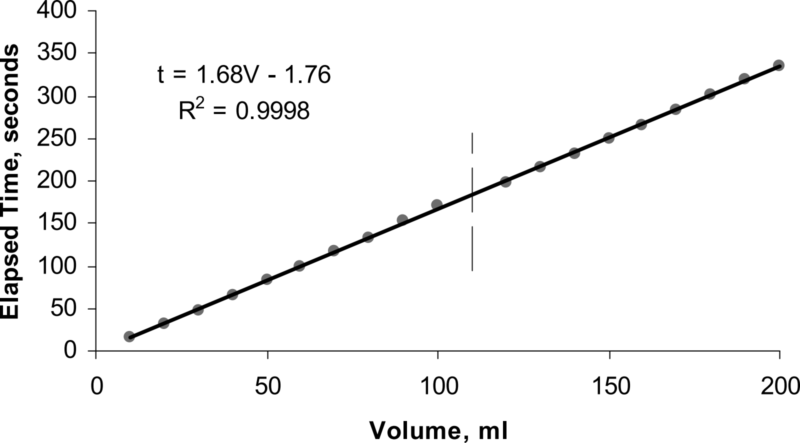
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